



Newton's Method for Roots of Functions

- activity developed by John Springall, Strathfield Girls H.S. for Jennifer Faber's Yr 11 extn 2 class

Newton's Method for finding the approximate root of a function is to repeat iterations of

$$a = x - \frac{f(x)}{f'(x)}$$

When the idea of using the TI-83 to illustrate this was first suggested, my immediate response was that a CAS calculator would be required. Then John developed an ingenious adaptation of our "function machine" idea to essentially build a "Newton's Method function machine".

The first step is to input the function under Investigation into Y_1 and then define Y_2 as the Newton's Method formula for this function. Note how this involves a division of the function value *at a point*, by the numerical derivative of the function at that same point.

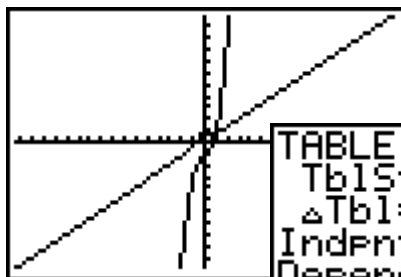
```

Plot1 Plot2 Plot3
\Y1=X^3+X-1
\Y2=X-Y1/nDeriv(
Y1,X,X)
\Y3=
\Y4=
\Y5=
\Y6=
    
```

nDeriv(is found in the MATH menu.

The TI-83 cannot directly produce a derivative function, but it can produce numerical derivatives for set points and this is all that is needed for Newton's Method!

At this point students should view the graph of Y_1 to gain some appreciation of where the root(s) lie. (Note that although Y_2 is also shown here, it's graph is not important to our illustration and you may choose to turn it off in the function editor.)



```

TABLE SETUP
TblStart=
ΔTbl=1
Indent: Auto
Depend: Ask
    
```

Now set the independent variable to "Ask" in TBLSET.

Display the TABLE (with Y_1 switched off if you wish) and commence with an input value in the neighbourhood of the root.

X	Y1	Y2
0	-1	1
1	1	.75
.75	.17188	.68605
.68605	.00895	.68234
.68234	2.9E-5	.68233
.68233	5.3E-6	.68233

X=

Continue to input the Y_2 output value until it converges to a value of acceptable accuracy.

Now that this is set up, you only need to change Y_1 to see how this method works for other functions.

X	Y2
-1	-1.222
-1.222	-1.194
-1.194	-1.194

Y1=X^4-5X-8

X=0 Y=-8

X	Y2
2	2.0741
2.0741	2.0697
2.0697	2.0697

Note: Of course the TI-83 always had the power to directly determine the roots using "zero" from the CALC menu. The purpose of this lesson, however, is to provide an illustration of the iterative nature of Newton's Method. Interestingly it is this method that is actually used by the calculator when the "zero" feature is employed.