Surprising findings in mathematics

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Manageable tasks for stage 4/5 in thinking mathematically
Radio frequencies

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What you will need

- A calculator
- A radio to check out the broadcasting frequencies of various radio stations

The Question

Radio station JJJ broadcasts on FM 105.7 in Sydney What does this mean? How do FM stations differ from AM stations?

Background information

Frequency Modulation (**FM**) and Amplitude Modulation (**AM**) are two different methods of radio broadcasting. When you tune into a station on your radio you tune in to the broadcasting frequency. A frequency of 1 cycle per second is called a **Hertz (Hz)**.
For example, ABC JJJ is on FM 105.7. This means that the radio wave being transmitted from ABC JJJ is 105.7 Megahertz, that is 105.7 million Hertz, or 105.7 million cycles per second.

The overall band of radio waves varies between about $3x10^3$ Hz and $3x10^{11}$ Hz. Radio waves in the **AM** band are usually measured in the **kilohertz (KHz)** range. They have lower frequencies than those in the higher **FM** band, which are measured in **megahertz (MHz).**

$$1 \text{ KHz} = 10^3 \text{ Hz} = 1\ 000 \text{ Hz}$$
$$1 \text{ MHz} = 10^6 \text{ Hz} = 1\ 000\ 000 \text{ Hz}$$

High frequency FM radio signals have good immunity to electrical interference and provide consistent quality reception during the day and night. However FM radio reception can be difficult when travelling in built up areas or in undulating terrain. AM radio signals of medium frequency follow the contours of the ground. They travel large distances and can achieve very wide coverage areas. For example ABC NewsRadio in Sydney broadcasts on 630 AM, that is 630 KHz. It is a national AM station, while Triple J is a national FM station.

Frequency and wavelength

Radio waves, just like microwaves, x-rays and other forms of electromagnetic radiation travel at **the speed of light (c** m/s). They have a **wave length** λ the distance from peak to peak of the wave measured in metres, and a **frequency f** which describes the number of cycles it makes per second. The wavelength of a radio wave is its speed divided by its frequency.

$$\lambda = \frac{c}{f}$$

Learning activities

1. The speed of light, denoted by c, is approximately 300 million m/s. So for an FM station broadcasting in megahertz (millions of Hz) the above equation becomes

   $$\lambda = \frac{300 \text{ x } 1\,000\,000}{\text{f x } 1\,000\,000}$$

   $$\lambda = \frac{300}{f} \quad \text{.................. (1)}$$

   Use equation (1) to show that the wavelength of the radio wave broadcast by (Sydney)ABC JJJ 105.7 FM is 2.8 m (to 1 decimal place)

2. For an AM station broadcasting in kilohertz,

   $$\lambda = \frac{300 \text{ x } 1\,000\,000}{\text{f x } 1000}$$

   $$\lambda = \frac{300000}{f} \quad \text{................ (2)}$$

   Use equation (2) to show that the wavelength of the radio wave broadcast by ABC Newsradio 630 AM is 476.1 m (to 1 decimal place)

3. How many times greater is the wavelength of Newsradio 630 AM than JJJ 105.7 FM?

4. How many times greater is the frequency of the radio wave of ABC JJJ 105.7 FM to Newsradio 630 AM?

5. What do you notice about the answers to questions 3 and 4? Why does this happen?

6. Fill in the missing words: As the wavelength of a radio wave gets longer, its frequency ……………. As the frequency of a radio wave increases the wavelength gets _____________ . The frequencies of FM stations compared to AM stations are generally much ____________. However radio waves from both AM and FM stations travel at the ________________________________.

7. Draw a diagram to show the difference between high frequency short wave radio waves, and low frequency long wave radio signals.

8. Have you learned something that has surprised you? Write briefly about it here.

Challenge questions

AM radio stations in Australia operate on two frequency levels, Medium Frequency (MF) and High Frequency (HF).

1. MF broadcasting relies on radio waves which are transmitted radially (in every direction) from the station and follow the contours of the earth. The frequency band of MF radio is from 531 KHz to 1602 KHz. Calculate the equivalent band in wavelengths.

2. HF broadcasting, also known as “short wave” radio, transmits in the frequency band from 2000 KHz to 26000 KHz. For example the ABC’s Radio Australia short wave broadcasts have reached people in Asia and the Pacific for many years.
   (a) Express the frequency band for short wave radio in MHz.
   (b) Calculate the equivalent band in wavelengths.

3. Draw the MF and HF bands on a number line to show the range of their wavelength and frequency.

4. If v is the speed of sound, the wavelength λ of a sound wave is related to its frequency by the relation

$$\lambda = \frac{v}{f}$$

Given that the speed of a sound wave is 343 m/s, calculate the frequency of the sound wave whose wavelength is 30 cm.

5. The immense coverage area of short wave (HF AM) radio broadcasts is achieved by transmitting the radio waves upwards. These *skywaves* are then reflected downwards by the *ionosphere* (the upper layer of the Earth's atmosphere) to a planned area on the Earth's surface. Use the internet to find out
    a. How high is the ionosphere?
    b. How does the reflection work? Why not use a satellite?

Teaching notes

1. It is advisable to work through the activities first before deciding if your class ready for it. It was designed and trialled successfully with year 8 students in their second year of high school in NSW after a unit on speed, distance and time.

6. The activity can be very motivating to students and will get them thinking. It is a good example of an inverse relationship. As one quantity increases, the other decreases. A diagram will help develop understanding of the relation $\lambda = \frac{c}{f}$.

   Teachers should note that for any propagating wave, sound, light, X-rays or other electromagnetic forms, $\lambda f = v$ the velocity of the wave. Thus for a **sound** wave, the constant v becomes the speed of sound (about 343 m/s at 20º C). Teachers may wish to use this for further challenge questions.

7. The mathematics includes: applying indices in an unfamiliar context; studying an inverse relationship; substituting into formulae and interpreting answers; using a calculator to evaluate an algebraic expression; applying knowledge about speed and distance in an unfamiliar context – see PAS 4.4 and 5.1 page 86-87 Years 7-10 syllabus.

8. It is not intended that the activities get too scientific. The basic idea is simply to get students interested in radio broadcasting as an application of speed = distance per unit time relationship. A radio in the classroom will be of advantage. A useful web page for teachers to prepare can be found at http://www.abc.net.au/reception/radio/listen.htm . A good web site to explore the challenge questions can be found at http://www.radio-electronics.com/info/propagation/HF/ionoprop.php and for information about the ionosphere http://www.haarp.alaska.edu/haarp/ion1.html

Answers to learning activities

Q3: 168 times greater
Q4: 168 times greater
Q5: They are the same. Wavelength and frequency are in an inverse relationship. The ratio of the longer wave to the shorter wave is equal to the ratio of the higher frequency to the lower frequency.
Q6: decreases; smaller; higher; speed of light

Answers to challenge questions

1. 565m to 187m (nearest m)
2. (a) 2 MHz to 26 MHz (b) 150m to 11.5m
3. Insert diagram here
4. 1143 Hz
5. (a) 80km to 190km (b) Bouncing radio waves off the ionosphere is very cost effective and efficient. However satellites are more reliable.

Links to working mathematically keywords

- Explaining
- Applying strategies
- Interpreting answers
- Reasoning
- Questioning
- Reflecting
- Communicating
- Searching for further information