MANSW Conference 2006

Manageable tasks for stage 4/5 in thinking mathematically

A mysterious number hidden in a secret code

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What you will need

- A calculator

The Question

What number is hidden in the symbolic code

Insert continuous fraction here

1 + 1 .
    1 + 1 .
        1 + 1
            1 +

Background information

In his story **The da Vinci Code** Dan Brown writes about a mysterious code, or hidden message, which Leonardo da Vinci (1452 – 1519) painted into his famous works of art. The book concerns the attempts of Robert Langdon, Professor of Religious Symbology at Harvard University, to solve the murder of renowned curator Jacques Saunière of the Louvre Museum in Paris. The title of the novel refers, among other things, to the fact that Saunière's body is found in the Louvre naked and posed like Leonardo da Vinci's famous drawing, the *Vitruvian Man*, with a cryptic message written beside his body and a pentagram drawn on his stomach in his own blood. The interpretation of hidden messages inside Leonardo's famous works, (which relate to the concept of the Sacred feminine) including the *Mona Lisa* and *The Last Supper*, figure prominently in the solution to the mystery.

The mathematician Leonardo of Pisa (1170-1250) was born in Italy 300 years before Leonardo da Vinci. Both shared a love of mathematics. In this activity we will investigate a mathematical mystery which Leonardo of Pisa saw in a famous sequence which is still known by his nickname. It is called the Fibonacci sequence.

Learning activities

Surprising findings in mathematics

1. Complete the next five numbers of the Fibonacci sequence
   1,1,2,3,5,8,…………………….

2. Fibonacci knew that this sequence has many mysterious properties. Here is one of them. Form a new sequence by dividing each term by the one before it. Complete the next five terms of the sequence

   1/1, 2/1, 3/2, 5/3, …………………………

3. Starting from the first term 1, plot the members of the sequence on the number line below. What do you notice?

4. Make a conjecture (a reasoned guess) about the number towards which the sequence appears to be approaching.

   (a) Could it be a whole number? Explain why/why not.
   ……………………………………………………………………………………
   (b) Could it be an improper fraction? Explain why/why not.
   ……………………………………………………………………………………
   (c) What is its approximate value as a decimal?
   ……………………………………………………………………………………

The mystery number

The number towards which the sequence is approaching is hidden in the symbolic code mathematicians call a *continued fraction*. If we call it n,

n = 1 + 1/(1 + 1/1 +1/1 ….

1. What can patterns can you see in the continued fraction?
   ……………………………………………………………………………………
   ……………………………………………………………………………………
   ……………………………………………………………………………………
2. Let's now uncover the hidden number. Because the fraction continues to infinity we will have to work out the value of the hidden number step by step. Check the entries in the table for steps 1 to 4 and then complete the table to step 8. Approximate the improper fractions to 3 decimal places with a calculator.

Step	Fraction	Exact value	Decimal value
1	1	1	1
2	1+1/1	2	2
3	1+1/1+1	3/2	1.5
4	1+1/1+1/1+1	5/3	1.667

5			
6			
7			
8			

3. What do you notice about the value of the continued fraction as you increase the number of steps? Have you seen these values before? …………………………….
   ………………………………………………………………………………………
   ……………………………………………………………………………………..
4. Does the fraction appear to be getting closer and closer to a definite number? What might it be? ………………………………………………………………..

5. What is the connection to the Fibonacci sequence above?

   ………………………………………………………………………………………

Challenge questions

The mystery number hidden in the symbols of the continued fraction is called the *Golden Number* or the *Golden Mean*. The ancient Greeks knew of this number and considered it to have magical properties, in particular its geometrical properties. Go to the internet and find out

1. What symbol was given to the Golden Number
2. What a Golden *rectangle* is
3. How to draw a Golden *rectangle*
4. What is the connection of the Golden Number to Leonardo's *Vitruvian Man* and the *pentagram* drawn in blood on the murdered man in the Da Vinci Code?
5. Other famous artists who have used the Golden rectangle in their art and architecture?
6. What examples of the Golden Number are found in nature?

Teaching notes

1. This activity can be very motivating to students. Many will have heard of the Da Vinci Code, so the idea of a hidden number within a strange symbolic form will catch their interest. It is advisable to work through the activities first before deciding if your class is ready for it. It was designed and trialled successfully with year 7 and 8 students in high school in NSW.

2. In terms of working mathematically, the activity allows the student to apply their knowledge in an unusual context, make conjectures and test them, and think mathematically. The issue of proof may arise with advanced classes, so teachers should be ready to address this. Nothing has been proved at all in this exercise. It is really an exercise in inductive reasoning.

3. The mathematical content includes: fractions, decimals, reciprocals, sequences, plotting a sequence on the number line, and an intuitive idea of a limit. Teachers

may wish to introduce particular terminology to suit their class. The idea of a continued fraction may be outside most middle school syllabuses across Australian states. However our purpose here is simply to use the idea for a more important activity in thinking mathematically (see NS4.3 p63 Syllabus Years 7-10)

4. Teachers are advised to help their students where necessary with the evaluation of the continued fraction. The main idea is for them to see the pattern in the results and to make the connection with the Fibonacci sequence investigated before. It has worked well with stage 4 students but could be used for any appropriate class where teachers wish to apply student knowledge in an unusual context. Go to http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/cfINTRO.html#sqrtcf for a good introduction to continued fractions.

5. Teachers may note that continued fractions for irrational numbers like √2 or the Golden Number repeat. But not all continued fractions are infinite. Both proper and improper fractions terminate. Try it out with 13/8 for example. Divide out and write it as

   1+5/8 = 1+ 1/(8/5) = 1+1/1+ (3/5) = 1+1/1+ (1/(5/3)) = …..

6. Teachers may wish to plan follow up lessons to investigate the exact value of the Golden Number as (1 +√5)/2 with advanced classes. Useful internet sites include http://goldennumber.net/

7. Go to http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html for Fibonacci in nature. Type *Golden number* into Google and click on *images* to explore wonderful examples in art, architecture, nature, etc..

Answers to learning activities

1. 1,1,2,3,5,8,13,21,34,55,89,……
2. 1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, …….
3. The sequence alternates and appears to converge towards a number which is approximately 1.618
4. (a) no; no whole number can lie between 1 and 2 (b) although it cannot be an improper fraction, students may argue, for example, that " it appears unlikely; the series goes on indefinitely so there will always be another improper fraction of the sequence closer to the answer." (c) 1.618 to 3 decimal places.

Answers to the evaluation of the continued fraction

The hidden number in the symbolic code emerges as we evaluate the continued fraction step by step.

Step	Fraction	Exact value	Decimal value
1	1	1	1

2	1+1/1	2	2
3	1+1/1+1	3/2	1.5
4	1+1/1+1/1+1	5/3	1.667
5	1+1/1+1/1+1/1+1	8/5	1.6
6	1+1/1+1/1+1/1+1/1+1	13/8	1.625
7	1+1/1+1/1+1/1+1/1+1/1+1	21/13	1.615
8	1+1/1+1/1+1/1+1/1+1/1+1/1+1	34/21	1.619
9	1+1/1+1/1+1/1+1/1+1/1+1/1+1/1+1	55/34	1.61765
10	1+1/1+1/1+1/1+1/1+1/1+1/1+1/1+1/1+1	89/55	1.61818

Links to working mathematically keywords

- **Conjecturing**
- **Explaining**
- **Looking for a pattern**
- **Reasoning**
- **Communicating**
- **Applying strategies to solve a problem**
- **Searching for further information**