Winning Hearts and Minds:
A New Look for the Extension 2 Course

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The Backdrop

- Extension Mathematics syllabuses have not changed substantively in over 20 years
- During this time we have seen Mathematics re-invent itself as a discipline
- Consistent “brain drain” of our best and brightest Year Twelve Mathematics students to “high status” tertiary courses in Business, Law and Medicine
- Interest in tertiary courses in Science and Engineering has waned in the last decade
- Continued supply of quality teachers of Mathematics is in grave jeopardy

The Extension 2 Status Quo

- The current course has a decidedly "pure bent" (Barrington & Brown, 2005)
- Some topics are unnecessarily intimidating (even for good students), difficult to motivate and viewed as "hurdles to leap" rather than "knowledge to keep"
- University students draw a disturbing distinction between their courses and "School Mathematics"
- Opportunities to portray Mathematics as a vibrant, evolving, exciting and **applied** discipline are minimal
- Some topics in the current Extension 2 course are unlikely to help us win the "battle of hearts and minds"

An Important Perspective

- Secondary Schools must provide students with the potential and ambition to undertake a science/engineering degree, with programs in Years 11 and 12 to ensure they have adequate preparation to readily progress to science and Mathematics studies in the first year of university. Additionally, part of the education budget should specifically be allocated to training teachers capable of fostering science and engineering in schools. (FASTS Workshop report, 2004, p105).

What is Needed

1. Collaboration

"The key to raising standards and developing excellent Year 12 Mathematics syllabuses and associated assessment practices nationally is to support collaboration between outstanding Year 12 teachers with experience in these areas along with [University] mathematicians"

(G. Gaudry cited in Barrington & Brown, 2005)

2. More opportunities to model, with a focus on ODE's.

"Among all mathematical disciplines the theory of differential equations is the most important ... It furnishes the explanation of all those elementary manifestations of nature that involve time."

Marius Sophus Lie (1897)

An Example Unit of Work: Extension 2 Projectiles

- See Coutis, P. F. (1998) Modelling the Projectile Motion of a Cricket Ball, *International Journal of Mathematical Education in Science and Technology*, 29(6), 789 – 798.
- Aims:
  - To extend current work to include more realistic scenarios
  - To develop in students a greater appreciation for the power and utility of mathematical modelling
  - To encourage deep thinking about the relationships between abstract Mathematics and the phenomena it describes
  - To improve understanding of the properties of known functions, and differential calculus
  - To stimulate interest and engagement by combining theory and practice

The BIG Question(s)

- **Why do we assume no air resistance?**

- When is that a valid assumption?
- What would happen if we included drag?
- What would the Mathematics look like?
- Could we still solve it?

Modelling with Drag

- In reality, drag force a complicated function of instantaneous velocity
- Will choose a linear term for simplicity and "fit" to data
- Equations of motion:

$$m\ddot{x} = -c\dot{x}$$
$$\Rightarrow m\ddot{x} + c\dot{x} = 0$$
$$\Rightarrow \ddot{x} + k\dot{x} = 0 \quad [1]$$

and

$$m\ddot{y} = -c\dot{y} - mg$$
$$\Rightarrow m\ddot{y} + c\dot{y} = -mg$$
$$\Rightarrow \ddot{y} + k\dot{y} = -g \quad [2]$$

- Can't integrate directly!
- $\boldsymbol{Q}$: What sort of functions might work?

Use What You Know!

- Derivatives of function must combine to give zero or constant
- Try $x = e^{mt}$
- Then $\dot{x} = me^{mt}, \ \ddot{x} = m^2 e^{mt}$
- Substitution gives $e^{mt}(m^2 + km) = 0 \Rightarrow m = 0, \ m = -k$
- So $x(t) = Ae^0 + Be^{-kt} = A + Be^{-kt}$ (Note: Using linearity property)
- $\boldsymbol{Q}$: How do we find A and B?

Apply the Initial Conditions

- For launch from the origin $x(0) = 0, \ \dot{x}(0) = v_0 \cos\theta$
- This gives $A = -B = \dfrac{v_0 \cos\theta}{k}$
- Finally $x(t) = \dfrac{v_0 \cos\theta}{k}\left(1 - e^{-kt}\right)$
- cf $x = (v_0 \cos\theta)t$ for zero drag case. Note $t = 0$ case in both.
- $\boldsymbol{Q}$: What happens to $x(t)$ as $t \rightarrow \infty$?

Motion in the Vertical

- For launch from the origin $y(0) = 0, \quad \dot{y}(0) = v_0 \sin\theta$

- More difficult (since d.e. non-homogeneous) but can show that

$$y(t) = \left(\frac{v_0 \sin\theta}{k} + \frac{g}{k^2}\right)\left(1 - e^{-kt}\right) - \frac{gt}{k}$$

- Rearranging x-equation gives $t = -\frac{1}{k}\ln\left(1 - \frac{kx}{v_0 \cos\theta}\right)$

- Cartesian form (on eliminating t – good algebraic exercise!)

$$y = x\left(\tan\theta + \frac{g}{kv_0 \cos\theta}\right) + \frac{g}{k^2}\ln\left(1 - \frac{kx}{v_0 \cos\theta}\right)$$

Comparing Trajectories …

- With zero drag:

$$y = x\tan\theta - \left(\frac{gx^2}{2{v_0}^2}\right)\sec^2\theta$$

- With linear drag:

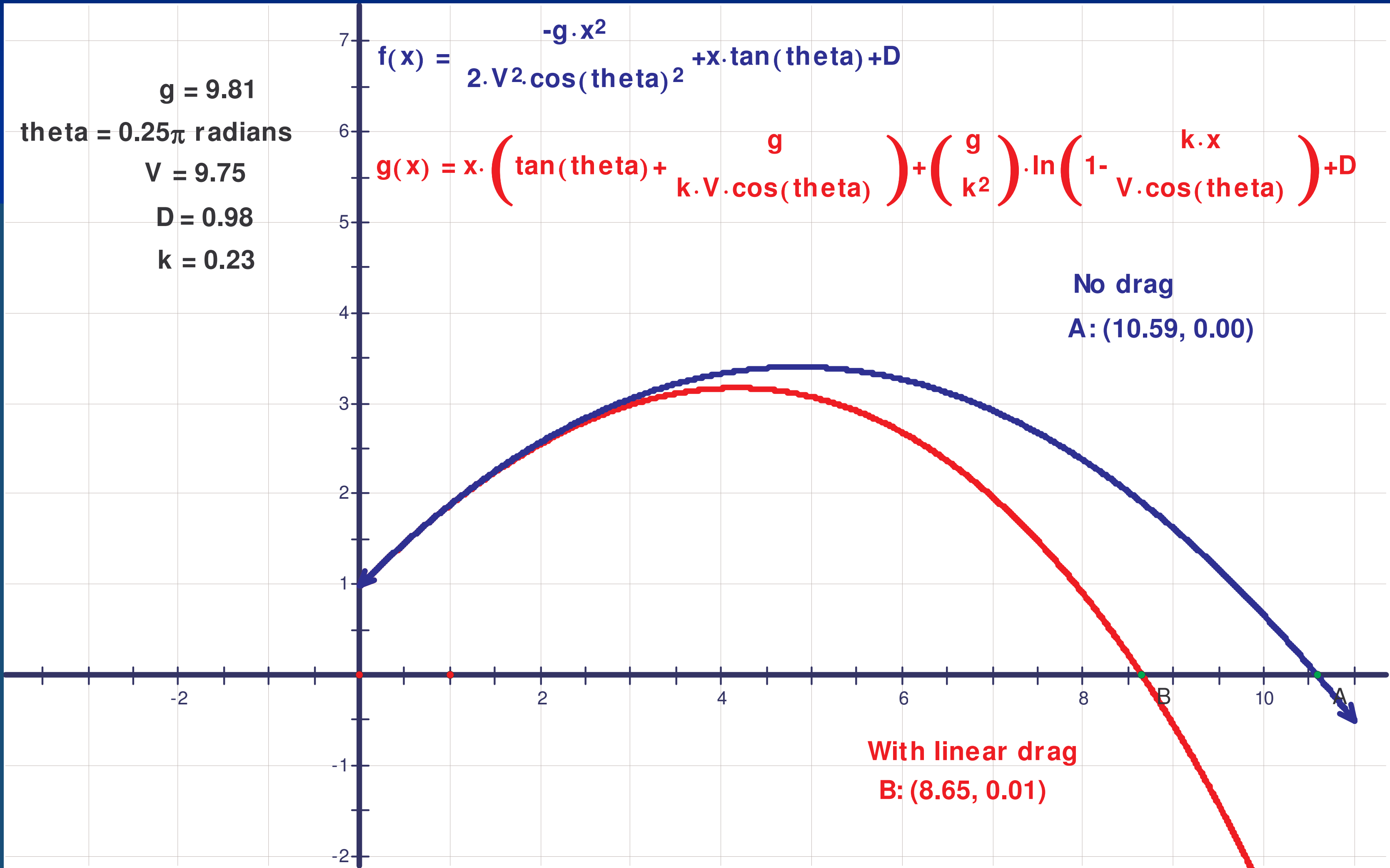
$$y = x\tan\theta + \frac{gx}{kv_0\cos\theta} + \frac{g}{k^2}\ln\left(1 - \frac{kx}{v_0\cos\theta}\right)$$

- ***Q***: For a fixed value of *x*, is $y_{\text{drag}} > y_{\text{no drag}}$??? What the?

- Note also: $y \to -\infty$ as $x \to \frac{v_0\cos\theta}{k}$ which means …?

Numerical Comparisons of Trajectories

- Sketchpad to the rescue …



Experimental Set-up

- Equipment required: projectile launcher, retort stands, light gate, data logger (and PC), sand bucket, tape measure, stop watches.
- Long, flat, wind-protected space (i.e. firing range!)

Experimental Data (SI Units)

Launch Height	Launch Angle (°)	Launch Speed	Range	Predict (no drag) % Error	Predict (drag) % Error
0.98	46	9.75	8.68	10.59 22%	8.66[fit] 0.2%
0.98	60	10.24	7.21	9.79 36%	7.60 5%
0.98	80	10.44	3.08	3.97 29%	2.97 4%
0.98	80	8.81	1.74	2.87 39%	2.24 29%

Student Feedback

What did you like most about the projectiles unit?

- Physically able to see projectile motion $\rightarrow$ leads to a better understanding
- Being involved in the process, we all participated …
- That what I've learnt is actually quite applicable, stimulating "Real World" situation instead of "Perfect HSC World"
- Applicable to everyday life, interests me and complements Physics
- Showing how it was applicable to real life, proving that the equations actually have a meaning
- Shows practical component to the theoretical, very important, see it in "action". Practical and theory balance very important.

Suggestions for the "New Look"

- OUT:
  - Conic Sections
  - Extension 2 Polynomials
  - Harder 3 Unit topics
- IN:
  - Mathematical modelling with ordinary differential equations
    - First order (separable, linear, homogenous)
    - Second order (restricted cases)
  - A range of applications to Medicine, Engineering, Mechanics, Biology, Industry
  - Technology to support modelling and exploration

Benefits of a Modelling Focus

- Enhanced interest and engagement
- Greater opportunities to scrutinise student thinking
- Stronger links to tertiary study and improved preparation for same
- A natural forum for the <u>meaningful</u> use of technology
- Greater scope for (in School) assessment
- A more realistic presentation of what Mathematicians DO with Mathematics and greater interest in Mathematics as a viable tertiary study option
- Improved opportunities to attract highly able Mathematicians to teaching?

Giving Students What They Want

- A modelling approach will give us increased opportunities to **give students what they want**
  - attributes of teachers most valued: enthusiasm, ability to stimulate thinking, knowledge of subject, tying information together (e.g. Broder and Dorfman, 1994)

and avoid what they don't want

  - courses perceived as boring, hard, and irrelevant discourage higher level study in Mathematics (e.g. Brown, 1999)

The Final Words …

- A metaphor for change – see "The Battle of Geticeberg"

- "Don't be afraid to take a big step if one is indicated – you can't cross a chasm in two small jumps".

David Lloyd George