Equations Instruction: A comparison of Balance and Inverse Methods

Bing Hiong Ngu

Albury High School

Structure of Presentation

- Cognitive Load Theory
- Equations Instruction: Balance and Inverse methods
- Experiment 1
- Experiment 2
- Conclusion

Rationale

- Equations pose great challenge to most high school students.
- NSW maths textbooks advocate the use of Balance Method to learn equations.
- (see Kalra & Stamell, 2005; McSeveny, Conway, & Wilkes, 2000).
- Research into instructional design from the cognitive load theory perspective

Cognitive load theory

- Cognitive load theory (Sweller, 1988, 1989, 1994; Sweller, van Merrienboer & Paas, 1998)
- Three aspects of cognitive load:
  - extraneous load
  - intrinsic load
  - germane load

Extraneous load

It occurs when learners are required to process instructional activities (e.g., search, integrate, relate pieces of elements to become intelligible) unrelated to skill acquisition.

- Convention problem solving and worked examples (Cooper and Sweller, 1987)
- Integrated worked examples and conventional worked examples (Tarmizi and Sweller, 1988)

Intrinsic load

- It is concerned with the inherent demands of the learning tasks which impose different cognitive loads upon the learners depending on their expertise in a domain.

  Direct the learner to engage in a sub-component rather than the whole task (Gerjets, Scheiter & Catrambone, 2004) will reduce intrinsic load

German Cognitive Load

- It occurs when the learner's attention is directed to engage in cognitive activities that profit skill acquisition.

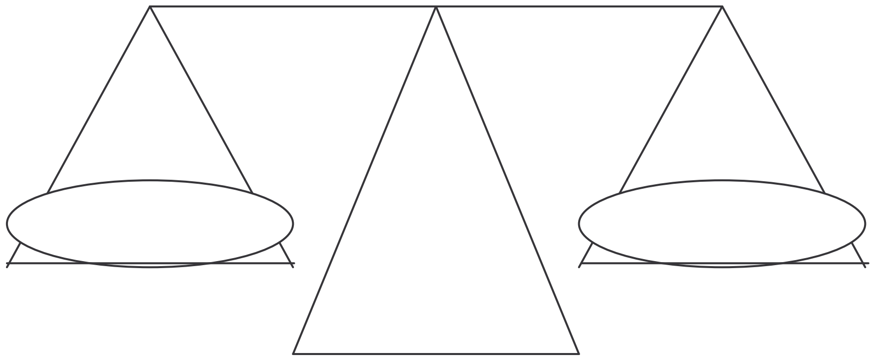
    Attempting to self-explain solution procedure of worked examples will incur additional mental load which is germane or beneficial in nature.

Limited cognitive resources

Learning will only occur if the total cognitive load (intrinsic, extraneous, and germane load) stays within the memory resources of the learner (see Paas, Tuovinen, Tabbers and Van Gerven, 2003).

Balance method

$x + 2 = 5$



3 + 2 = 5

x = 3 balances the scale

So, x = 3 is the solution

One-step equations (Balance method)

1. $x + 2 = 5$ (- 2) on both sides

$$\begin{aligned} x + 2 &= 5 \\ -2 \quad & \quad -2 \\ x &= 3 \end{aligned}$$

2. $4a = 12$ ($\div$ 4) on both sides

$$\begin{aligned} 4a &= 12 \\ \div 4 \quad & \quad \div 4 \\ a &= 3 \end{aligned}$$

Balance method: Summary

- Solving equations is like balancing scales.
- When we add (+), subtract (-), multiply (x) or divide (÷) both sides of the equation by the same number, both sides will remain balanced because the same operation is done to both sides.
- The solution of the equation is the value of the pronumeral that 'balances' the equation.

Extraneous load: Balance method

the learner searches (e.g., which operation to use?), and integrate pieces of elements (i.e., hold reverse operation on numbers) to make sense of the balancing of equation.

- write on both sides of the equation the reverse operation performed on each number for every solution step.

Inverse method: One-step equations

1. x + 2 = 5 (remove +2 from one side to

   x = 5 - 2 become – 2 on the other side)

   x = 3

2. 4a = 12 (remove x 4 from one side to

   a = 12/4 become ÷ 4 on the other side)

   a = 3

Inverse method: summary

- Use inverse operation (e.g., If +, then –) to work back to the pronumeral.

- Perform inverse operations to remove all numbers associated with the pronumeral.

- Like ‘peeling’ of an onion, remove the outer layer first (the number furthest from the pronumeral).

Less cognitively taxing: Inverse method

- Key concept is the use of inverse operation to remove each number in sequence associated with the pronumeral

- requires to write fewer lines of solution steps as compares to the Balance method.

Equations with pronumerals on both sides

Balance method

$$
\begin{array}{ll}
3x + 1 = 2x + 8 & \text{(-2x) on both sides} \\
-2x \qquad\quad -2x & \\
x + 1 = 8 & \text{(-1) on both sides} \\
\quad -1 \quad -1 & \\
x = 7 &
\end{array}
$$

Equation with pronumeral on both sides

Inverse method

$$3x + 1 = 2x + 8$$

$$3x - 2x = 8 - 1$$

$$x = 7$$

Experiment 1

Hypothesis:

Inverse method would be better than the Balance method

Participants:

- 25 year 8 students
- average maths abilities
- knowledge of one-step equation ($x - 3 = 8$)

Materials:

- Pre-test and Post-test
- Instruction Phase
- Acquisition Phase

Experiment 1 (continues)

Procedure:

- Session one: concept of inverse operations
- Session two: introduction of method
- Session three:
  - Pretest (10 minutes)
  - Instruction Phase (5 minutes)
  - Acquisition Phase (15 minutes)
  - Post-test (10 minutes)

Pre-test and Post-test

One-step equation

$m + 8 = 12$

Two-step equation

$5 + 3n = 10$

Instruction Phase:Balance and Inverse methods

- Aim of experiment
- Definition of an equation
- Study four worked examples of one-step equations presented in their respective methods
- Summary of solving an equation in their respective methods

Acquisition phase

- Students studied a worked example and then to solve a conventional equation (Sweller & Cooper, 1985; Cooper & Sweller, 1987).

- Students would expect to compare each pair of equations which shared a similar problem structure, and to transfer the method depicted in the worked example to solve the equation.

Acquisition Phase:Balance method

$3a + 2 = 8$ (- 2) on both sides

$- 2 \quad - 2$

$3a = 6$ (÷ 3) on both sides

$\div 3 \quad \div 3$

$a = 2$

Equation 1

$2m + 7 = 13$

Acquisition phase:Inverse method)

$3a + 2 = 8$ (+ 2 becomes - 2)

$3a = 8 - 2$

$3a = 6$ (x 3 becomes ÷ 3)

$a = 2$

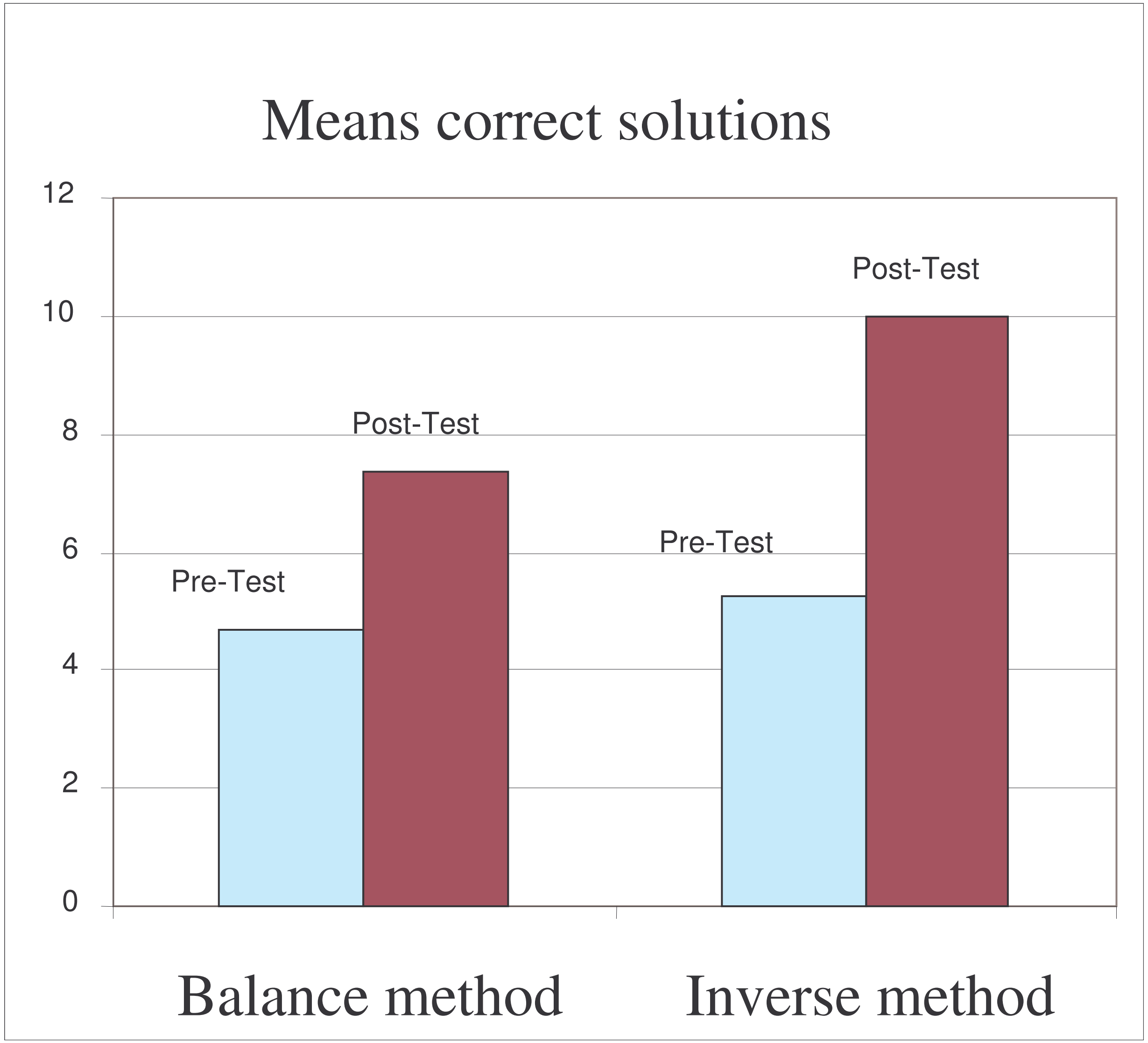
Equation 1

$2m + 7 = 13$

Experiment 1

	Inverse method	Balance method
	n = 13	n = 12
	M (SD)	M (SD)
Correct solutions		
Pre-test	5.27 (2.19)	4.71 (2.57)
Post-test	10.00 (3.21)*	7.38 (3.22)
Practice equations	7.19 (2.53)	9.17 (1.95)*

Experiment 1



Experiment 2

- Hypothesis: Inverse method would be better than the Balance method in facilitating learning of equations with pronumerals on both sides.

- Participants:
  - 27 year 9 students
  - Above average maths abilities
  - knowledge of one-step and two-step equations using Balance method

Experiment 2 (continues)

Procedure:

Single session

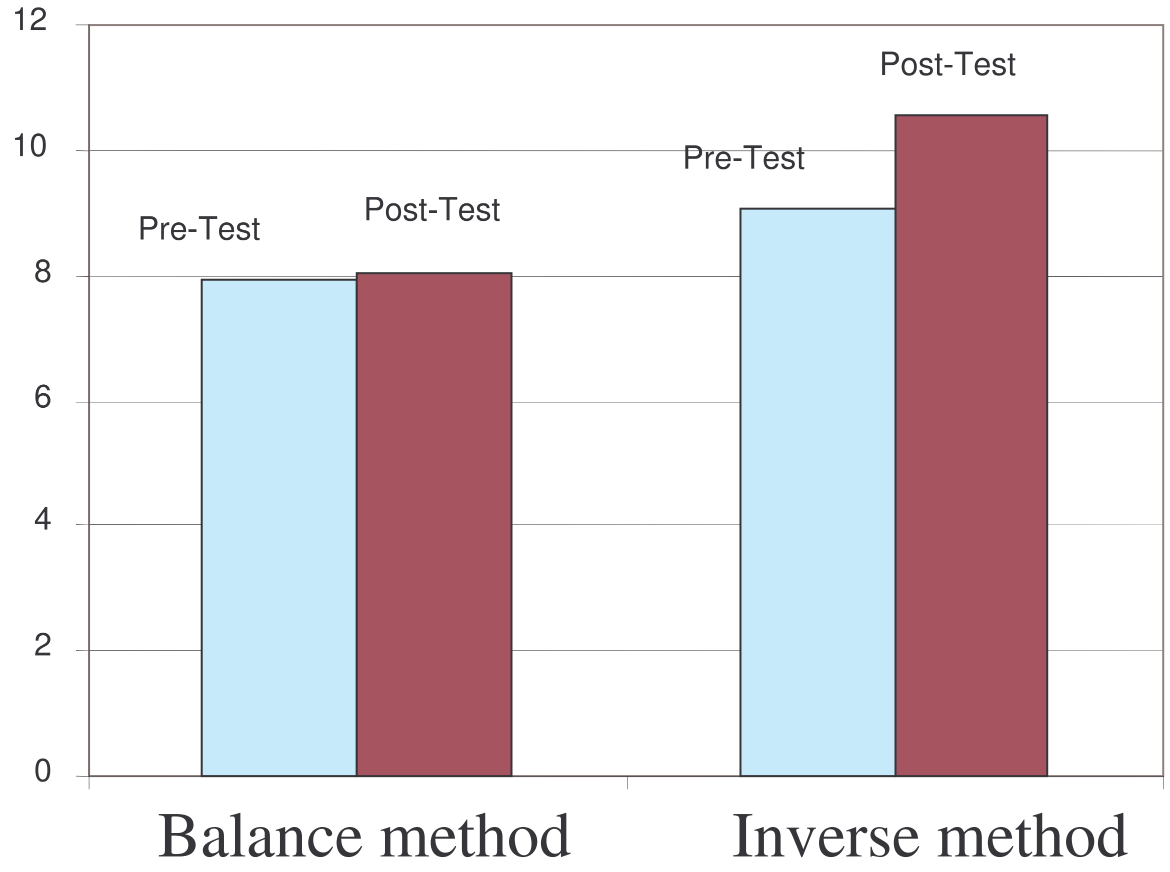
- Pretest (10 minutes)
- Instruction Phase (5 minutes)
- Acquisition Phase (15 minutes)
- Post-test (10 minutes)

Experiment 2

	Inverse method	Balance method
	n = 13	n = 12
	M (SD)	M (SD)
Correct solutions		
Pre-test	9.08 (2.99)	7.93 (4.14)
Post-test	10.58 (2.61)*	8.04 (3.76)
Practice equations	7.62 (2.60)	6.43 (2.82)

Experiment 2

Means correct solutions



Conclusion

- Inverse method is better than the Balance method in both experiments
- Extraneous cognitive load associated with the Balance method
- key concept of the Inverse method may incur less effort
- Larger sample of participants is required
- Need to measure cognitive load incurs during the acquisition phase
- Cognitive load theory is a useful tool to evaluate the suitability of equations instruction.