STAGE 6 MATHEMATICS

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> There is nothing like returning to a place that remains unchanged to find the ways in which you yourself have altered.
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> Nelson Mandela. *A long walk to freedom.* (1995)

With the exception of the General Mathematics syllabus released in 1999, the Stage 6 Mathematics syllabuses have changed little since 1982. Although the calculus-based Mathematics courses are unchanged, the nature and proportion of students enrolled in these courses have changed over the years. For example, there has been a noticeable reduction in the number of students taking only the Stage 6 Mathematics course since 2001 and an increase in the number of students not studying any mathematics in Year 12.

Some may believe that this is a good thing, but I do not. As Steen (1997)[1] points out, "…*as information becomes ever more quantitative and society relies increasingly on computers and the data they produce, an innumerate citizen is as vulnerable as the illiterate peasant of Gutenberg's time.* (p. xv) Considering the numeracy demands of active participation in society, the percentage of students currently not enrolled in any Mathematics course in Stage 6 is a concern. I believe that in the revision of Stage 6 Mathematics syllabuses it is important that the range of mathematics courses on offer encourages students to continue their education to Year 12.

There is a clear need to offer a breadth of curriculum that is relevant and vibrant within the Mathematics learning area to meet the full range of needs of students. The Stage 6 General Mathematics course with a current candidature of almost 30 000 students may need to be augmented by an additional Mathematics course, possibly addressing issues of quantitative literacy. It appears unreasonable to expect that this single course could adequately address the needs of the diverse interests of the majority of students completing Year 10.

One of the key findings of the recently published (May 2006) *Evaluating mathematics provision for 14–19-year-olds* (Ofsted, 2006)[2] was that having a "range of learning programmes that promote wider access to mathematics for students at all levels" made a significant contribution to high achievement of 14–19 year olds in mathematics in the United Kingdom.

The needs of society

One step towards addressing the low retention rates in Stage 6 is to provide Mathematics courses suitable for the needs and interests of all students completing Year 10. This will require recognising the needs of students entering a rapidly changing environment.

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[1] Steen, L. A. (Ed.). (1997). *Why Numbers Count: Quantitative Literacy for Tomorrow's America*. New York, NY: The College Board.

[2] Ofsted. (2006). Evaluating mathematics provision for 14–19-year-olds. Retrieved 15 May, 2006
www.ofsted.gov.uk/publications/index.cfm?fuseaction+pubs.summary&id=4207

At the macroeconomic level, Australia does not produce a sufficient number of statisticians to meet its own needs. Statistical expertise is an essential ingredient of high quality public health research and practice. For example, lack of adequate preparation in biostatistics (the application of statistics to the analysis of biological and medical data) appears to be an international problem.

Most of the key issues affecting day-to-day life are dependent upon mathematical and related statistical understanding. Avian influenza and the threat of a pandemic (epidemiology), national security (encryption, probability), transport timetables (networks, logistics), fuel costs, finance, insurance, banking, environment (carbon sinks, heating of the earth, tsunamis, weather forecasts), and most issues of public policy (aged care, superannuating) are dependent upon mathematics.

Yet at the school level, the undergraduate level and the university department level there is a reported shortage in mathematics. Over the past ten years, the Australian Mathematical Society estimates there has been a 25 per cent reduction in teaching positions in mathematics and statistics around Australia. Even the skill shortage areas, such as construction and electrical trades, rely upon the application of a wide range of mathematical skills and knowledge. At the same time that we have an increased need for a highly mathematically literate workforce, supply appears to be diminishing.

Course structure in Stage 6 Mathematics

The nature of Stage 6 mathematics courses should serve the needs of all Stage 6 students. That is, the courses should continue to prepare pure mathematicians as well as those drawing on mathematics methods in further study and those who seek simply to gain a richer appreciation of their world.

Replacing the General Course with a Standard and Advanced course

Although General Mathematics is well regarded as a practical alternative to the Stage 6 Mathematics course, it is a single course with an enrolment of just under 30 000 students. That is, this one course is attempting to meet the diverse needs of almost half of the HSC candidature. To promote wider access to mathematics for students at all levels, consideration should be given to developing two courses from the current General Mathematics course. For example, a course on *Applications of fundamental mathematics* could be built upon, through the provision of an *Advanced applications of fundamental mathematics* course.

Much of the content of an advanced applications of mathematics course could be developed from the current General course. For example, the applications to financial mathematics, the applications of trigonometry and spherical geometry, the applications of probability and algebraic modelling would all be relevant to this advanced course.

The fundamental applications course could emphasise using mathematics to understand significant social issues. By emphasising principles from mathematics, such as comparing like with like, the graphical display of data or the difference between causation and correlation, students could come to understand the logic at work in issues such as global warming or the role of immigration in Australia's population growth.

Stage 6 Applications of fundamental mathematics
The *Evaluating mathematics provision for 14–19-year-olds* (Ofsted, 2006) also identified that one of the factors acting against effective achievement, motivation and participation was "a narrow focus on meeting examination requirements by 'teaching to the test', so that although students are able to pass the examinations they are not able to apply their knowledge independently to new contexts, and they are not well prepared for further study". Many mathematics teachers would agree with this finding.

In designing a *Stage 6 Applications of fundamental mathematics* course, a small number of key principles should be used as the curriculum organisers. This should allow time to develop depth of understanding. Stage 6 mathematics courses build upon Stage 5 Mathematics. A *Stage 6 Applications of fundamental mathematics* course could build on Stage 5 Mathematics by using the techniques from the content strands, such as Measurement, Data and probability (within the Number strand), up to 5.1 in applications of basic mathematical principles.

For example, one principle might be *Comparing like with like.* A teaching activity addressing this principle, could develop from the following example.

> At the July 2001 conference on global warming at Bonn, Australia argued to use "carbon sinks" as credit for cutting greenhouse gas emissions. Carbon sinks are basically forests that eliminate carbon dioxide from the atmosphere.
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> A government report some years back indicated that the area of forest in Australia had increased from 43 million hectares in 1992 to just under 157 million hectares in 1998. How could this be? Taking the numerical data to be correct, what could account for this rapid increase in the growth of forests in Australia? What could have changed?
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> The current Australian Government Department of the Environment and Heritage website suggests that "a forest of trees with a potential height of at least two metres and crown cover of at least 20 per cent" and "in patches greater than one hectare in area" may be counted as afforestation. Are we in a forest?

Comparing like with like could be exemplified through a range of issues related to the environment, such as "Are hybrid cars environmentally friendly or fuel efficient?" Applications of mathematics related to health could be used to distinguish between causation and correlation.

> There was once a cholera epidemic in Russia. The government, in an effort to stem the disease, sent doctors to the worst-affected areas. The peasants of the province… discussed the situation and observed a very high correlation between the number of doctors in a given area and the incidence of cholera in that area (i.e. more doctors were observed in cholera areas than elsewhere). Relying on this hard fact, they rose and murdered the doctors. Fisher, F. M. (1969). *The Identification Problem in Econometrics*, pp. 2-3.

> Explain, with reference to causation and correlation, what the problem was? Indicate how you could reason that the doctors weren't causing the cholera.

Stage 6 Advanced applications of fundamental mathematics
A *Stage 6 Advanced applications of fundamental mathematics* course could be based upon the structure of Stage 6 General Mathematics and extend the algebraic modelling component. For example, instructions for cooking a leg of beef suggest that you should allow 55 minutes for each kilogram plus 20 minutes in a gas oven temperature of 180° C. Set up an algebraic model suggested by this information and describe the limits that should be set on the variables used in the model. How could you improve the model?

In this constantly changing world, those who can reason mathematically will have significantly enhanced opportunities and options for shaping their futures.

A separate statistics course
The place of statistics should be considered carefully in the revision of the Stage 6 Mathematics courses. It may be possible to design a separate 2 Unit statistics course around exploratory data analysis, so that students could take 2 Units of Mathematics and 2 Units of statistics. Such a course could be a non-calculus statistics course based on exploring data, sampling and experimentation, exploring random phenomena using probability and simulation, and statistical inference.

Alternatively, an Extension 1 Statistics course could build on a common 2 Unit Mathematics course as an additional option to Extension 1 calculus. An Extension 1 Statistics course could include work on conditional probability (e.g. Bayes theorem), the law of large numbers (Bernoulli's theorem), variance, applications of Chebyshev's inequality and perhaps the Chi-squared distribution.

Although the design of a statistics course should take place as part of the revision of the Stage 6 mathematics syllabuses, it might be advisable to look at the first examination of the course being two years later than the other Mathematics courses. This should enable adequate time to address issues of teacher supply and training.

It has been ever thus, or has it?
In the nature of my work I often encounter discussions about the teaching of the basics. This sometimes takes the direction of comments on the teaching or drilling of multiplication tables in schools. In these discussions, which frequently refer to a golden age [that never existed] when everyone knew times tables, I am struck by the lack of appreciation of the history of mathematics. Although we joke that things have not changed greatly in the teaching of mathematics over the years, they have. Consider the following anecdote.

> It appears that a [German] merchant had a son whom he desired to give an advanced commercial education. He appealed to a prominent professor of a university for advice as to where he should send his son. The reply was that if the mathematical curriculum of the young man was to be confined to adding and subtracting, he perhaps could obtain the instruction in a German university; but the art of multiplying and dividing, he continued, had been

> greatly developed in Italy, which in his opinion, was the only country where such advanced instruction could be obtained.
> Dantzig, T. (1954). *Number: The language of science.* New York: Macmillan & Co.

My use of this anecdote is to clarify the link between the needs of society and what mathematics is taught in schools. Society and the corresponding needs of society change over time and so too do the mathematics courses taught in schools.

The Stage 6 Mathematics syllabuses have much that is worthwhile preserving, and perhaps that is why they have remained so long. However, I am reminded of a Hebrew proverb:

> ***Do not confine your children to your own learning, for they were born in another time.***