NSW BOS: Stage 6 Mathematics Symposium, 19 August 2006

New lamps for Old? John Mack.

The NSW Stage 6 pattern of courses in Mathematics has been stable for a quarter of a century, apart from a short-lived course introduced at the request of the then Minister for Education "to address the needs of those students for which the present courses are not suitable". *Mathematics in Practice* went out with the McGaw review, because its intended outcomes were, quite correctly, identified as below the expected outcomes of Year 10 Mathematics. But for which group of students were these 'expected outcomes' developed? The spread of mathematical achievement across a year 10 cohort is expected to be from years 7 to 12+ levels. On this basis, then, the current Stage 6 courses probably suit a smaller proportion of Stage 6 students than they previously did. This may be a partial reason for the larger proportion of HSC students now studying no mathematics, but it is likely not to be the only reason for this.

A reasonable number of high UAI students do not study an HSC mathematics course. I do not know exactly why this is so, but I doubt that it is because they lack the ability to perform well in Mathematics. Rather, I suspect that their own interests and future intentions, at least at the time they chose their HSC courses, led them to their choice. Similarly, I suspect that those who did study a given HSC Mathematics course did so for the same reasons. But, post HSC completion, some UAI-eligible students will undoubtedly want to alter their post-school plans for reasons other than "don't waste your UAI!". This has a bearing on university admissions criteria and I will discuss this later.

The spread of attainments and interests at Stage 6 level certainly also supports strongly my view that we must try to ensure that some of our Maths courses will both stimulate those with mathematical ability and also encourage them to develop that ability further post the HSC, either in a career path or as a lifelong interest. Given the numbers currently enrolled in Extensions 1 and 2, who continue their study of Mathematics at tertiary level, together with the numbers in those courses who choose other tertiary level programs of study, it is clear that courses which stimulate able and highly able Stage 6 students to include Mathematics among their subject choices must continue to be offered. In fact, given the recent delayed recognition that Australia is facing a severe shortage of engineers and scientists, to say nothing of mathematicians and statisticians, we should try to ensure that all courses in Stage 6 Mathematics attract those taking them into post-secondary courses that use, or build on, these courses. This should be just as applicable to those in Stage 6 who study VET courses, or who will be in school apprenticeships, or who intend to pursue vocational courses outside higher education. I suggest also that it is time for the total Stage 6 curriculum to examine a program of study that embraces and continues the *Life Skills* offerings in 7-10 as well as potentially being a bridge into some topics in the mathematics courses.

I should at least mention the possible impacts of the current national ACE/KCA initiatives on Stage 6 Mathematics. A consistent approach by all governments to lifelong learning addressing national needs and also individual aspirations does require the maintenance and preferably the enhancement of current curriculum continuity, diversity, and cumulative learning across all levels of education and training, for all citizens. Simplification and some standardisation of course descriptions, in terms of content, knowledge and skills, and comparable measures of outcomes, and agreement on a common core of these in each major subject area, seem to me to be sensible ways of enabling cross- border utilization of the considerable

amount of effort currently devoted to the ongoing, dynamic, monitoring and development of learning programs at all levels.
But I do not know of any guaranteed, one-size-fits-all curriculum at **any** level of Mathematics and so I am totally opposed to any attempt to impose a common curriculum, teaching plan, or assessment type, at all, unless there is evidence in support of a particular model as excelling above all others. Meanwhile, we have evidence suggesting that what we are able to do at present is reasonably satisfactory for a reasonable number of Stage 6 students. I will make a comment below about the proposed KCA.

Other state/territory educational authorities will also have similar evidence, and yet may differ greatly in their educational programs. This is the case in Mathematics, but I do not really believe that the present differences seriously impede cross-border movement of students either at Stage 6 or immediately after it, into different learning programs. For my part, I think that decisions to change the structure of our Stage 6 Maths courses, or their interrelationships, or their intended educational outcomes, must be based on well-informed understanding of the resulting value, to the individual and to the nation, of the new course program, balanced by a pragmatic understanding of those factors which will determine the **delivered** curriculum so that it in fact does resemble the **published** curriculum, as measured by student outcomes.

Key factors here are: the teaching staff and the available resource base at school level, the same two items in terms of their availability globally via one or other form of student accessible learning environments, and opportunities for students to do their own explorations via accessible resources, whether they be others at school or in the community, or via the media, or online. I do not see that, in this age of information overload and accessibility, we can cheerfully reassure our students of the universality of mathematics in everyday life and work, unless we can expose them to real-world examples via the media.

'Aha!', I hear you mutter, " and where is a real world example of x and y, quadratic expressions, coordinate geometry, or the calculus?" There is no easy answer, and finding answers will become more and more difficult with each new use of technology in our lives. Why so? Because new technology is accepted if it enables us to do something better, or new, or more easily than is available to us at present. Moreover, more and more people want access to usable, better technology, so much design effort goes into keeping technology simple. In particular, the mathematics used in the design, or the manufacture, of technology is, more and more, hidden from view to the user, and, in addition, much technology is devoted to reducing the level of mathematical competence needed by the end user!

Take, for example, the cash register, originally designed to record a sales amount and keep money secure. The user had to do all the associated financial transactions. Today, a serious cash register will not only keep a record of all sales, but itemise the sale items, deduct these from an inventory register, handle cash and cards, and advise the change. In the case of a ticket machine, it will allow you to choose from a large selection, give the price, and automatically dispense the change.
A financial maths package will do all calculations related to banking and investment, or, eg, to loans and mortgages and give you a full accounting of the results.
In a different area, satellite technology gives a GPS that enables you to use in your car a dynamic road map in more and more locations, thus removing the need for any use of a street directory and its associated coordinate system, while CAD packages can

now give you a solution to just about any structural design problem once you have the input specs. The use of finite-element-method packages in engineering is universal, except for designs that have become off-the-shelf.

Does all this mean that we really don't have to know about the underlying design of the technology, or the mathematical, scientific and engineering principles underlying that design? If we want to make a new piece of technology, or improve on an existing piece, certainly not! If we want to have some way of testing the output of a piece of technology, we have to develop a 'feel' for what that output should be. When the output gives a numerical value, we had surely better have a safe way of estimating what that value should be, and preferably with a good understanding of using order of magnitude approximations as a quick check. Hardly any of this is feasible without some understanding and experience of the mathematical processes used, in their relevant context, and of their outcomes.

At the level of 'ordinary applications of mathematics', then, we need to design a curriculum that recognises which 'contexts' are relevant to our future students and also how we might give them understanding and experience in the uses of those contexts, having taken into account the K-10 curriculum considered from this perspective, and having identified the new contexts most appropriate for development and/or desirable reinforcement in Stage 6.

We then need to apply a similar process when considering more specialised contexts, in which a progressive, or cumulative, introduction of newer or deeper mathematical ideas, knowledge and skills is desirable. I am thinking here of in-school, or post-school learning paths that will be feasible, or assisted, by the successful exposure of students in Stage 6 to other new, or deeper topics.

Finally, we need to apply a similar perspective to those contexts which will best encourage and assist those students who discover a pleasure in the ongoing learning of mathematics, or who appreciate how it can be understood at the various levels of abstraction that provide a basis for deeper conceptual understanding of the subject and its potential for high-level application in other subject areas.

One question to ask, in evaluating the contexts that emerge as candidates for inclusion in the curriculum, is whether or not we have the current staff and material resources to provide both any necessary PD to their teachers, and also the appropriate learning environments for the desirable teacher/student/other types of interactions.

A second question to ask , in terms of course structure, is how best to fit these contexts together into feasible course offerings, utilising linked contexts wherever possible, but also recognising that some contexts may lend themselves more readily than others for integration into a single course.

In summary, I ask for a Stage 6 Mathematics course design which has what we believe is an optimal structure and curriculum in terms of the best selection of contexts that will BEST serve the Stage 6 cohort, from the perspective of national need and individual aspiration. (Of course, I will have to modify this if at any time it turns out that our national needs are considered not to include an ever growing pool of the mathematically competent and the mathematically able.)

What do I want to say about T&L? Simply that, from my own learning experiences, and my understanding of the experiences of a large (and I mean large) number of students at university, there is value in encouraging both behaviourly automatic and deep conceptual learning **appropriate to each context.** I emphasise the latter, because deep learning at one level may need to be automated to enable a re-internalizing of concept development to occur. Mental arithmetic is still, I believe, useful in everyday life, and much easier to apply than using technology in many situations. Understanding that the processes of mental arithmetic are procedures based on conceptual understanding of certain operations on numbers, and when to apply them, is most likely a prerequisite for doing the relevant mental arithmetic computation. Learning about structural properties of, eg the integers, is facilitated by an ability to 'do the sums' without thinking, while considering features of the outcomes (eg, primes vs composites and the multiplicative structure of composites). Understanding how algebraic and numerical symbols relate and are used together, either in an algebraic or an analytical context, requires a careful development of our automatic computation skills to facilitate our learning of new concepts and structures, and so on.

I have seen my students suddenly have that flash of insight, which comes from getting on top of a procedure, or understanding the connection between their formal conceptual representation of a topic and a new one derived from the introduction of a new idea or the application of a concept learnt in a different context, to a current context. I have no doubt that mathematical ability often exhibits itself by the way in which a person will accept and integrate a piece of mathematics into their own internal representations of it, appropriate for different contextual levels, so deepening understanding of the mathematics and often enhancing the relationship between procedural and conceptual knowledge. I guess I am at heart, a social constructivist, using my understanding of that descriptor.

What suggestions do I offer for course content and design in the new structure? Based on my experience in mathematics teaching and on Admissions Committees at the University of Sydney, and also from experiences interstate and internationally, I would like to see:

- **A sensible connection, especially at the Extension course level, to entry level tertiary courses offered to attract good students and to provide smooth transition into academically demanding award courses, especially those requiring mathematics units in first and later years.** For example, the transition at these levels works smoothly under current arrangements, and it is a relatively simple provision to offer bridging courses (especially Mathematics to Extension 1) using current course content.
- **A steady development of the notions of function and their geometrical and algebraic representations, with useful examples including polynomials, exponential and trigonometric functions.**
- **Maintenance of a reasonable level of introduction to the calculus and to algebra, hopefully giving some automated skills and some deeper understanding of the physical concepts underlying the derivative as rate of change and the definite integral as an aggregate sum.** (Certainly, students interested in refining approximations towards a limiting value might be given help, but I don't expect that deep understanding of the necessary properties of the underlying number set will be an outcome except for a few.)
- **Development of applications in a number of different contexts, especially in the context of behaviour of functions and maybe with less need to**

include several from mechanics, but covering the modeling of dynamical phenomena in several different fields, with time spent on understanding how each context leads to a mathematical model.

- **A study of complex numbers and their geometrical representation; solving some simple cases of roots of unity; showing that if a real polynomial has a complex root Z then the complex conjugate of Z is also a root and from these one obtains a real quadratic factor; mention the FTA, contrast this with the problem of solution by radicals.**
- **Some two and/or three dimensional geometry.**
- **Some combinatorial probability and the binomial theorem.**
- **An introduction to cryptography, up to understanding the relative frequency of letters in English words and the means found for hiding this in some ciphers.**

Why have I chosen the final three above? Well, some 35 years ago, two dimensional Euclidean geometry was removed from the Years 7-12 syllabi, and soon after that, academics in several fields realised just how essential a knowledge of the basic 2 and 3 dimensional objects, and their simplest properties (such as symmetries) really was. (I think the most frequent single complaint to me was "They don't know that if two circles touch, the tangent at the point of contact is perpendicular to the line of centres!".) Since a generation of teachers had entered into schools without this knowledge, it was decided to put some relevant material back into 9-10 and also into 11-12, to enable them to learn it along with their students. It is now timely to take into account the new 7-10 syllabus and to decide how this might sensibly be utilized and extended via one or more topics in years 11-12, but I would not like to see a pathway through 11-12 that does not consistently enable basic geometrical facts to be reinforced and used.
The remaining two topics are related and allow extension of material developed in 7-10. I mention the exploration of some simple ciphers (call them 'codes' if you want to cash in on Dan Brown) because they provide a real use of probability. In fact, one of the principal Japanese codes broken in WW2 turned out to be fallible to a probability argument.

I am not sure about introducing matrices and vectors. I found that introducing linear algebra and vector spaces to second year university students of average mathematical ability was a challenge both to them and to their teacher, while using matrices to describe transformations in 3D required careful presentation at third year level to similar students. Perhaps, via say linear equations in two (and maybe) dimensions, one could describe how the solution of homogeneous and inhomogeneous equation systems can be linked, and doing so would offer opportunity to help students visualize the relative location and intersection possibilities of planes and lines.

I must comment on the topics of statistics and data analysis, for which my remarks below on the use of technology will apply in spades. I think there is value in continuing to offer learning and training in these two topic areas. Whether we like it or not, they are becoming more necessary, and more intrusive, into the skills we need to take an intelligent approach to understanding our lives, our communities and the turbulent world around us. We need to help our students understand how quantitative evidence can be used to distort, support, or obscure an argument, and also just to help us get a feel for the magnitude of problems around us. To do this well probably means that we need to encourage the understanding and use of the basic principles of logical argument-surely an across-the-curriculum task, and provide many examples taken from the media for study and analysis.

I think there may be a possibility for the current Mathematics course in Stage 6 to be examined with a view to identifying those topics which provide support for Extension 1 in Years 11-12, and those which might, possibly with a new topic mix, offer an alternative strand in Year 12 that covers topics in statistics and data analysis using good software and uses available high quality data bases (like Census data, etc).

Finally, some comments on assessment and technology. Mathematics and its language is both an extension of ordinary language and a technology-the latter because it assists us in communicating, expressing, and thinking about ideas not easily managed without it. The development of technologies to facilitate ‘ordinary’ language communication is fascinating in itself (think only of writing and the tools to do so, or of the technology developed enabling the use printing or typing, and then the codes used to convey messages when the telegraph and the wireless were invented, and the technology enabling speech transmission and then image transmission, etc. etc.).

Now think of the technologies that were developed to enable mathematics to be more easily done and communicated, and how much more difficult that was to do! I think of the very early word processing packages that could handle the usual mathematical symbols found in secondary mathematics (the early 1980s 2/3/4 unit syllabus documents were prepared using a package called T-cubed in the Department of Pure Mathematics at the University of Sydney, so they could be offset printed rather than typeset at great cost.) Now everyone uses packages derived from Donald Knuth’s *Tex* system.

Finally, think of the technologies that have been developed, and continue to be developed, to facilitate exploration and work in mathematics or its applications. Tallying devices and coins, mathematical tables, slide rules, arithmetical, scientific, and graphical calculators, and computer packages for exploration, special usage and general usage.
The most technically challenging of these have been the symbolic algebra packages, which utilize the full language of mathematics to facilitate calculation in and exploration of problems in various mathematical structures and contexts, both numerical and symbolic – eg, *Mathematica*, and the *Magma* system, developed under Professor John Cannon and colleagues at the University of Sydney and elsewhere, especially for use within a wide range of algebraic and number theoretic structures.

I cannot think of any area of work, from deep thought to digging holes in the ground, where one would resile from utilising the best available technological aids, except for the reason that one could not afford to do so. In some cases, one would borrow funds to do so, while in other cases, one would keep using ‘old’ technology because it remains the best suited for the task in hand, while waiting for something better to come along (or designing it oneself).

I think the same approach should apply to the use of technology in mathematics, whether it is needed for educational, practical, exploratory or research purposes. So, if we have determined an expected set of outcomes from a planned sequence of activities, and so have a context in which these will occur, what would be the best technologies we could use to facilitate the realisation of the outcomes? Answering this question does NOT mean that the most high-powered technology must be the most appropriate for a given context, but it does mean that we need to have a paradigm for enabling us to decide what 'best' means in each chosen context. Aye, there's the rub. Is there agreement on the best technological contexts as we progress K-12? I doubt it, because there are very disparate views on the likely impacts of technologies on learners, as well as disparate views on how one learns and is supported in learning. So, it may be an undecidable problem to define 'best' here. But that should not stop more research on this important matter, especially at Stages 4, 5 and 6.

For example, I would like to see a group of highly able students in late Stage 5 and throughout Stage 6 exposed to suitable smart computational and symbolic algebra packages in topics in the syllabus and around it. (One topic might be the RSA crypto system and how one can compose and decode messages sent in a simple but non-trivial model of it.)

With regard to assessment then, first build formative assessment processes into curriculum design, and offer them as support materials if they can't appear in the syllabus document, and do summative assessments to measure progress or as training for a small number that will count towards a final performance result. School-based assessment should, I think, continue to be used in determining the final result, along with external assessment measures, such as those used in the current HSC. I would like to see Year 11 remain free of 'final assessment' processes.

With regard to HSC examination paper structure, then, in the expected equivalents of current Extension 1 and 2 courses, I would like the Review to consider a redesign of the present papers, reducing the number of questions, and including one or two more exploratory extended tasks that utilise a combination of knowledge, skills and technology to solve, or explore possible solutions to, a problem that is formulated reasonably precisely but not so as to produce the automatic switching on of a 'this must be a context X problem' response.

Given the experience in Victoria in the 1990s, over attempts to introduce time-extended projects and problems into the classroom as part of the 'final assessment', which resulted in accusations of copying, cheating, buying solutions in the Victoria Markets, etc, then I concede that neither of these types of assessment are likely to become regular formative tasks until we find a means of using them summatively under controlled conditions.

There is one useful form of learning that I would like to see encouraged, and that is learning in groups. I think that this could be made more common across the curriculum by developing formative assessment criteria to help teachers provide advice to individual students, and to groups together, on ways of facilitating the operation of and outcomes from group work. Improving the performance here would even give satisfaction to those wanting more emphasis on "Employability Skills", and this leads me into a final comment on the current ACE/KCA proposals.

Do you remember the national project on key competencies K-10? The outcome of this project was a report identifying what became known as the Mayer Competencies, after the man who stimulated this activity. These were adopted by the Ministers and, if you look, eg, in the current NSW Mathematics 7-10 Syllabus document, you will see them identified as being incorporated into various strands of the syllabus.

This is what I believe should happen to those capabilities suggested in the ACE report as possibly being a basis for a new independent assessment task at Stage 6. Let us first decide which such capabilities are worth pursuing, and then investigate how to incorporate them into syllabus and assessment requirements ACROSS THE CURRICULUM. For example, the sorts of skills mentioned in the International Assessment programs investigated in the Report include, in mathematics, a number that easily fit into the contexts of other HSC subject areas, and in my view are likely to be more effective there. By this I mean that we should take the entire K-10 Mathematics program, and then insert elements of its outcomes, as appropriate, into every other subject's contexts right up to end HSC level, so giving opportunity for students to use applications in subject context and not just in a 'subject' context transferred into the Mathematics curriculum. By doing so, we may then legitimately say to our Stage 6 students that they will find applications of what they are currently learning in many post-secondary contexts, and have available a real list of such contexts.

Returning to my title: technology has replaced a lot of old lamps by new ones and has often been able to do this only because of major improvements in the infrastructure. Let us measure the improvements in the educational infrastructure, decide for what purposes we want our lamps, and then work towards producing a practicable and productive outcome. I wish the present review team well and look forward to the next range of lamps and may there be a genie in each one!

30/7/06.